Intraday Cross-sectional Distributions of Systematic Risk

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Intro

- Linear factor models: workhorse of empirical asset pricing

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- Probably not and we already know that
  - Extensive literature showing how moving betas help pricing the cross-section
  - They might change with firm characteristics: Fan et al. (2016), Kelly et al. (2019)...
  - Also with macro variables: Shanken (1990)

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  - They might change with firm characteristics: Fan et al. (2016), Kelly et al. (2019)...
  - Also with macro variables: Shanken (1990)
- An alternative: use data sampled at higher frequencies - betas are second moments!
  - Barndorff-Nielsen and Shepard (2004), Andersen et al (2005), Mykland and Zhang (2009)

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- We document similar behavior for other factors studied in the literature
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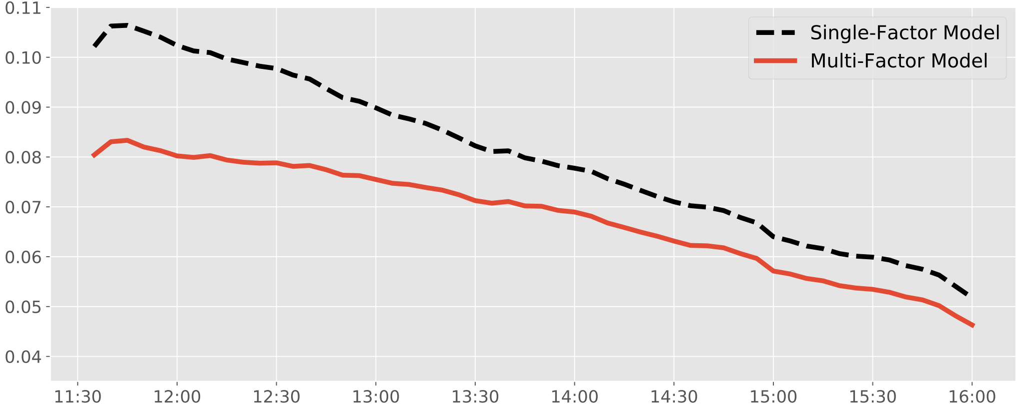
- We document similar behavior for other factors studied in the literature
- We link changes in cross-sectional distributions of loadings to information releases
- New inference tools to analyze the cross-section of betas in higher frequencies:
  - Allows for an arbitrary number of factors
  - Allows for latent orthogonal factors
  - Does not rely on a long panel
  - Uses characteristic functions: power against wider set of alternatives

Illustration - Dispersion of Market Betas

Figure: Cross-sectional variance of market betas at each point in time



- Compute market betas using backward-looking 2-hour windows pooling all days together
- Compute the cross-sectional variance of loadings

Illustration - Individual Betas

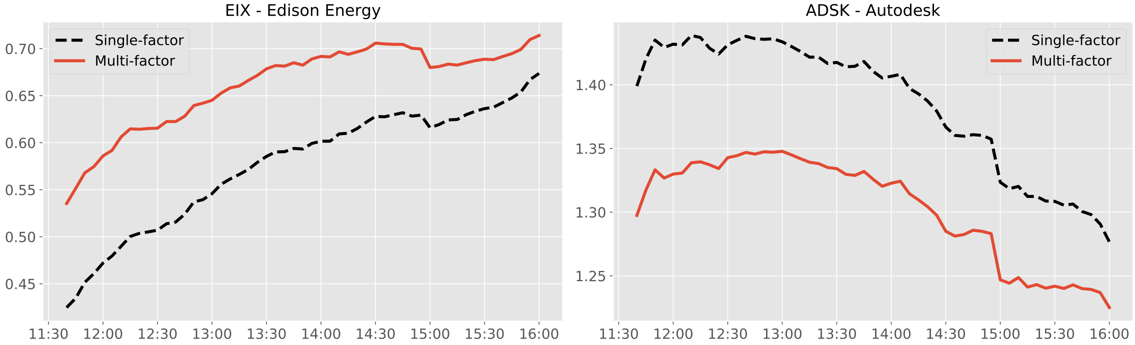


Figure: Cross-sectional dispersion of individual betas along the trading day

Methodology - General Model

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A Flexible Multi-Factor Model in Continuous Time:

$$F_t = F_0 + \int_0^t \alpha_s \, ds + \int_0^t \sigma_s \, dW_s + \sum_{s \leq t} \Delta F_s, \quad F_t \in \mathbb{R}^q \tag{1}$$

$$\begin{aligned} X_t^{(j)} = X_0^{(j)} &+ \int_0^t \alpha_s^{(j)} \, ds + \int_0^t \beta_s^{(j)\top} \sigma_s \, dW_s + \int_0^t \gamma_s^{(j)} \, dB_s \\ &+ \int_0^t \widetilde{\sigma}_s^{(j)} \, d\widetilde{W}_s^{(j)} + \sum_{s \leq t} \Delta X_s^{(j)}, \qquad j = 1, .., N, \end{aligned} \tag{2}$$

Methodology - Estimation of Loadings

- For any moment along the day $\kappa \in [0,1]$:
  - $\mathcal{I}_\kappa^n \equiv$ estimation window within a day
  - $k_n \equiv$ number of returns in $\mathcal{I}_\kappa^n$
  - $n \equiv$ number of observed returns within a day
- For any day t:

$$\widehat{V}_{t,\kappa} = \frac{n}{k_n} \sum_{i \in \mathcal{I}_\kappa^n} \Delta_{t,i}^n F \Delta_{t,i}^n F^\top 1_{\{\mathcal{A}_{t,i}\}}, \qquad \widehat{C}_{t,\kappa}^{(j)} = \frac{n}{k_n} \sum_{i \in \mathcal{I}_\kappa^n} \Delta_{t,i}^n X^{(j)} \Delta_{t,i}^n F \, 1_{\{\mathcal{B}_{t,i}^{(j)}\}}, \qquad (3)$$

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- Let $\mathcal{T}$ be a collection of trading days and define:

$$\widehat{V}_{\mathcal{T},\kappa} = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \widehat{V}_{t,\kappa}\,, \qquad \widehat{C}^{(j)}_{\mathcal{T},\kappa} = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \widehat{C}^{(j)}_{t,\kappa}\,, \qquad \widehat{\beta}^{(j,k)}_{\mathcal{T},\kappa} = \iota_k^\top \, \widehat{V}^{-1}_{\mathcal{T},\kappa} \, \widehat{C}^{(j)}_{\mathcal{T},\kappa}\,. \tag{4}$$

Methodology - Characteristic Functions

- We can define the cross-sectional characteristic function estimator for any $u \in \mathbb{R}$:

$$\widehat{\mathcal{L}}^{N}_{\mathcal{T},\kappa,k}(u) \;=\; \frac{1}{N}\sum_{j=1}^{N}\exp\left(iu\,\widehat{\beta}^{(j,k)}_{\mathcal{T},\kappa}\right), \quad \kappa \in [0,1], \quad k = 1,\ldots,q, \quad u \in \mathbb{R}. \tag{5}$$

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- As $n \to +\infty$ we measure loadings with higher precision
- As $N \to +\infty$ we estimate the characteristic function more precisely, for each u
- The cardinality of $\mathcal{T} \subset \mathbb{N}$ is fixed (doesn't need to be sequential)
- Theorem 1 is a functional CLT for $\widehat{\mathcal{L}}^{N}_{\mathcal{T},\kappa,k}$ - see the paper for details

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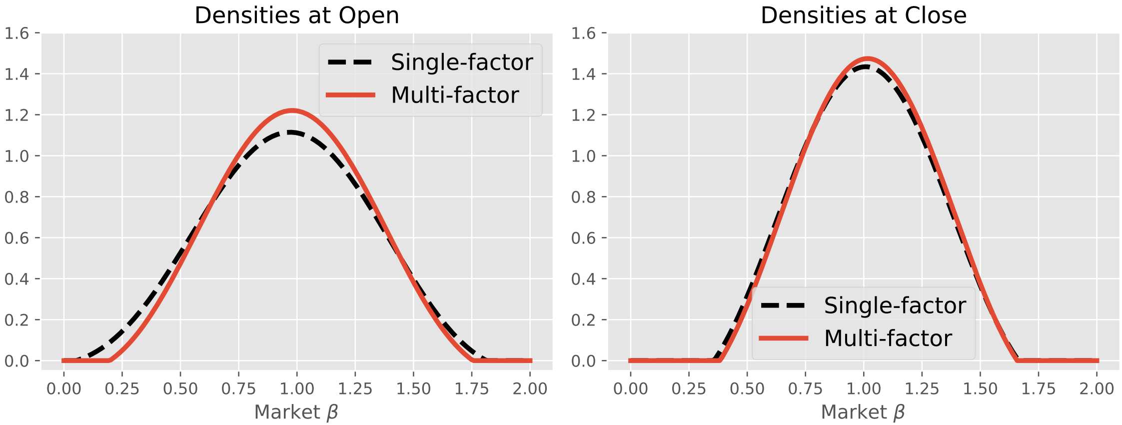
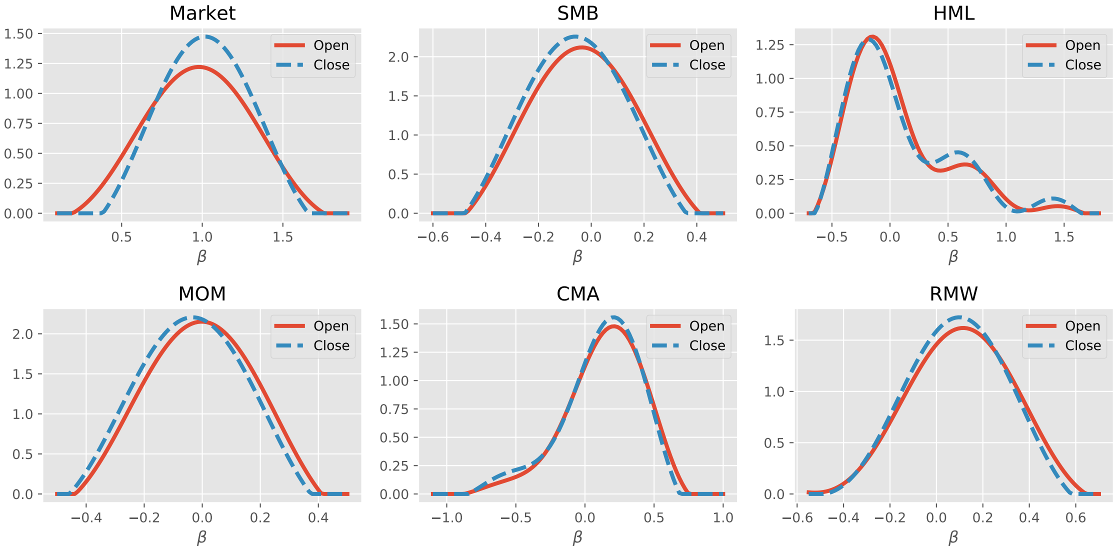
- The Inverse Fourier Transform will give us cross-sectional densities!

Densities for Market Beta - First and Last Two Hours of Trading



- Betas are more concentrated around the mean at Close
- Allowing for many factors has greater impact at Open

Densities for Other Factors - First and Last Two Hours of Trading



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- Our real-valued test statistic with a Gaussian $w(.)$:

$$TS_{\mathcal{T},\kappa,\kappa',k} \ = \ k_n \, || \, \widehat{\mathcal{L}}^N_{\mathcal{T},\kappa,k} - \widehat{\mathcal{L}}^N_{\mathcal{T},\kappa',k} \, ||_w \tag{7}$$

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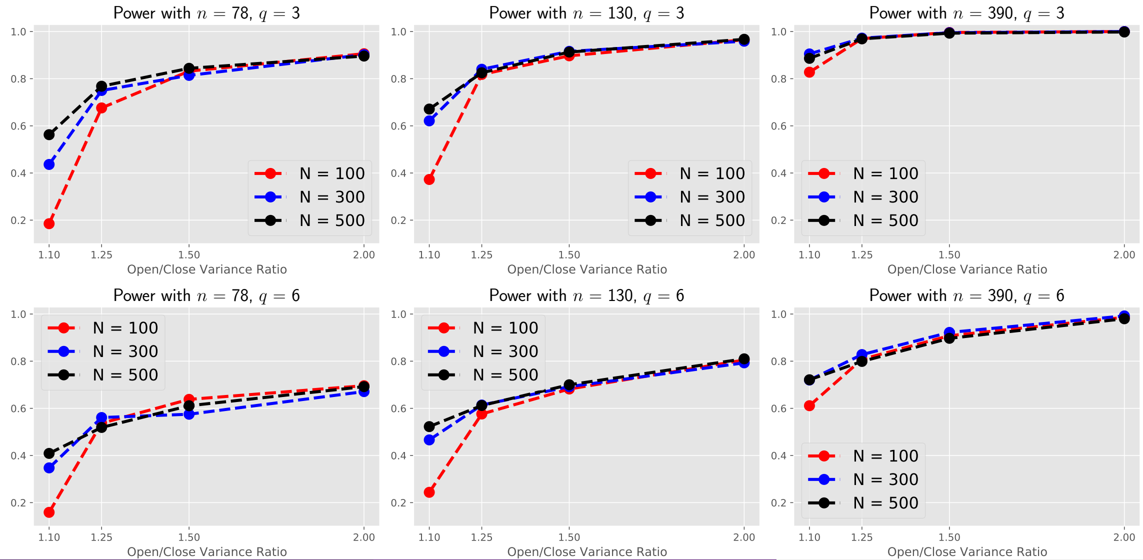
- The limiting distribution is complicated $\to$ we use a novel bootstrap procedure
- Limiting distribution depends on realized path of volatility processes
- Boostrapped p-values take long to compute... but it can be improved with parallelization

Monte Carlo - Power Curves



Testing the Null

- Our null hypothesis: cross-sectional distribution of betas is the same at *Open* and at *Close*
- We use different subsamples $\mathcal{T}$:
  - Full sample: 1969 trading days, 2010-2017, all stocks in the SP500
  - Days with FOMC announcements: 62 days, information released around 2pm-2:30pm EST
  - Days from weeks with scheduled earnings announcements: 142 days, same stocks

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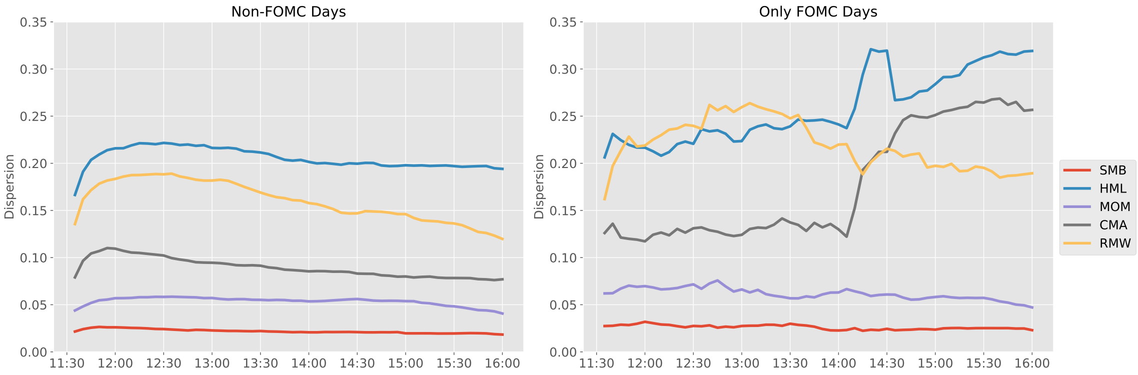
- The null for each of the 6 factors is tested independently of the other factors
- We use 2-hour estimation windows, 5-min sampling frequency
- All p-values are bootstrapped

Testing the Null: Full Sample and Monetary Policy Announcements

Table: Null hypothesis: same cross-sectional distribution of loadings at *Open* and *Close*

Sample/ Factor	Full Sample	FOMC Days	Non FOMC Days
Market	0.000	0.000	0.000
SMB	0.031	0.077	0.048
HML	0.000	0.001	0.000
MOM	0.199	0.196	0.214
CMA	0.603	0.019	0.483
RMW	0.014	0.915	0.015
Total Number of Days	1969	62	1907

Dispersion and Release of Information - Monetary Policy



- Dispersion changed is less pronounced on non-FOMC days
- But our test uses information about all moments, not only the second one

Testing the Null: Earnings Announcements

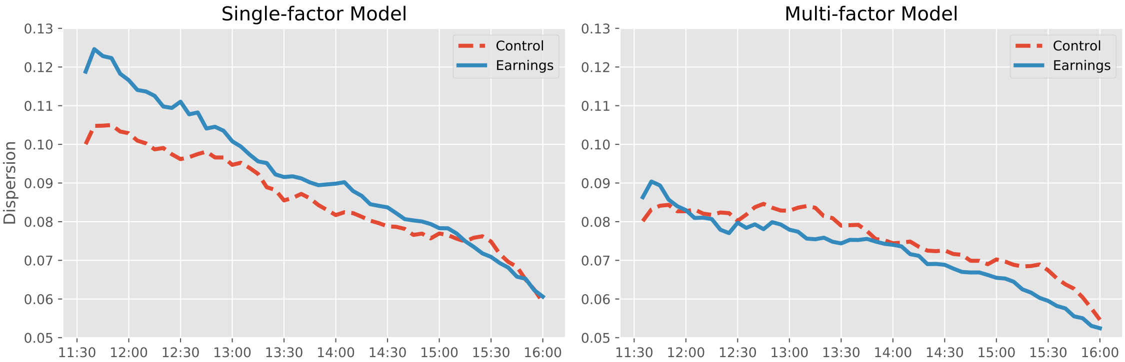
Table: Null hypothesis: same cross-sectional distribution of betas at *Open* and *Close*

Factor	Control Weeks	Earnings Weeks
Market	0.000	0.000
SMB	0.040	0.086
HML	0.088	0.035
MOM	0.263	0.097
CMA	0.380	0.332
RMW	0.993	0.009
Total number of days	142	154

- Control Weeks are the weeks before Earnings Weeks

Dispersion and Release of Information - Earnings Announcements

Figure: Dispersion of Market Beta - Control Weeks and Earnings Weeks



- Controlling for more factors reduces the morning gap - more information is spanned
- Even controlling for extra factors, the dispersion is monotonically decreasing along the day

Wrap-Up

Main messages:

- We document intraday variation of factor loadings using high-frequency data
- We formally test our findings using novel theoretical results
- Evidence about the market factor is very robust
- But controlling for extra factors seems quantitatively important for the puzzle
- Behavior of other factors seems related to specific information releases

Wrap-Up

Main messages:

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- Evidence about the market factor is very robust
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- Behavior of other factors seems related to specific information releases

Future Research:

- What is the implication for the risk premia associated to these factors?
- What are the most fundamental sources of intraday variation? Order flows? Information releases? Automatic rebalacing from ETFs?